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Deflationary cosmology: constraints from angular size and ages of globular clusters

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Observational constraints to a large class of decaying vacuum cosmologies are derived using the angular size data of compact radio-sources and the latest age estimates of globular clusters. For this class of deflationary $\Lambda(t)$ models, the present value of the vacuum energy density is quantified by a positive β parameter smaller than unity. In the case of milliarcsecond compact radio-sources, we find that the allowed intervals for β and the matter density parameter Ω_m are heavily dependent on the value of the mean projected linear size l. For $l \simeq 20h^{-1} - 30h^{-1}$ pc, the best fit occurs for $\beta \sim 0.58$, $\Omega_{\rm m} \sim 0.58$, and $\beta \sim 0.76$, $\Omega_{\rm m} \sim 0.28$, respectively. This analysis shows that if one minimizes χ^2 for the free parameters l, $\Omega_{\rm m}$ and β, the best fit for these angular size data corresponds to a decaying $\Lambda(t)$ with $\Omega_{\rm m} = 0.54$ $\beta = 0.6$ and $l = 22.64h^{-1}$ pc. Constraints from age estimates of globular clusters and old high redshift galaxies are not so restrictive, thereby suggesting that there is no age crisis for this kind of $\Lambda(t)$ cosmologies.

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I. INTRODUCTION

The search for alternative cosmologies is presently in vogue. The leitmotiv is the observational support for an accelerated Universe provided by the type Ia supernovae (SNe) experiments at intermediate and high redshifts [1, 2]. Such observations are consistently indicating that the bulk of energy in the Universe is repulsive and appears like a quintessence component, that is, an unknown form of energy (in addition to the ordinary dark matter), probably of primordial origin [3]. In the current literature there are many candidates for this negative-pressure dark component, among them: (i) a vacuum decaying energy density, or a time varying $\Lambda(t)$ [4], (ii) a time varying relic scalar field slowly rolling down its potential [5], (iii) the so-called "X-matter", an extra component simply characterized by an equation of state $p_x = \omega\rho_x$, where $\omega \geq -1$ [6] and (iv) models based on the framework of brane-induced gravity [7].

In a recent paper [8] (henceforth paper I), we derived the basic expressions for kinematic tests in a quintessence scenario of the type (i) above, which has originally been termed deflationary cosmology [9, 10]. The effective time dependent cosmological term is regarded as a second fluid component with an energy density, $\rho_v(t) = \Lambda(t)/8\pi G$, which transfers energy continuously to the material component. Such a scenario has an interesting cosmological history evolving in three stages. First, an unstable de Sitter configuration is supported by the largest value of the decaying vacuum energy density. This nonsingular state evolves to a quasi-Friedmann-Roberton-Walker (FRW) vacuum-radiation phase and, subsequently, the Universe approaches continuously the present vacuum-dust stage. The first stage harmonizes the scenario with the cosmological constant problem, while the transition to the second stage solves the horizon and other well-know problems in the same manner as in inflation. Finally, the Universe enters in the present accelerated vacuum-dust phase required by the SN Ia observations.

More recently, it has been argued that deflation can be described in terms of a scalar field coupled to the fluid component [11]. Such models are analytic examples of warm inflationary scenarios proposed by Berera [12] where particle production occurs during the evolution. As a consequence, the supercooling process, as well as the subsequent reheating are no more necessary. In this concern, it should be interesting to analyze the generation of the perturbation spectra by comparing the results with the ones recently obtained by Taylor and Berera for warm inflation [13]. Here, as there, one may expect a suppression of the tensor-to-scalar ratio perturbations, though a detailed study is necessary in order to have a more definitive conclusion. On the other hand, the scalar field in this enlarged framework is also thermally coupled with dark matter, and as such, it can be used to avoid the cosmic coincidence problem. Specific examples of exact deflationary models and the underlying thermodynamics has been given by Gunzig *et al.* [14], and some scalar field motivated descriptions were also investigated in detail by Maartens *et al.* [15] and Zimdahl [16].

In this paper we discuss more quantitatively how the observations put limits on the free parameters of the deflationary scenario. We focus our attention on constraints from the angular size of milliarcsecond compact radio sources, and the latest age estimates of globular clusters (GCs), and for completeness we also consider shortly the case for old high redshift galaxies (OHRGs). Next section we set up the basic equations for these models. In Section III we derive the corresponding constraints for deflationary cosmologies from $\theta(z)$ analysis. Limits from

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GCs and OHRGs are discussed in Section IV. The main results are summarized in the conclusion section.

II. BASIC EQUATIONS

Let us now consider a class of spacetimes described by the general FRW line element ($c = 1$)

$$ds^2 = dt^2 - R^2(t)\left[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

where χ, θ, and ϕ are dimensionless comoving coordinates, $R(t)$ is the scale factor, and $S_k(\chi)$ depends on the curvature parameter ($k = 0, \pm 1$). The later function is defined by one of the following forms: $S_k(\chi) = \sinh(\chi)$, χ, $\sin\chi$, respectively, for open, flat and closed universes. Since the main interest here is related to kinematic tests for deflation, in what follows we consider only the Einstein field equations (EFE) for a nonvacuum pressureless component plus a cosmological $\Lambda(t)$-term:

$$8\pi G\rho + \Lambda(t) = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} \quad , \quad (2)$$

$$\Lambda(t) = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} \quad , \quad (3)$$

where an overdot means time derivative and ρ is the dust energy density. The effective $\Lambda(t)$ is a variable dynamic degree of freedom. In our context, it relaxes to the present value, Λ_o, according to the deflationary ansatz [10]

$$\rho_v = \frac{\Lambda(t)}{8\pi G} = \beta\rho_T\left(1 + \frac{1-\beta}{\beta}\frac{H}{H_I}\right) \quad , \quad (4)$$

where ρ_v is the vacuum density, $\rho_T = \rho_v + \rho$ is the total energy density, $H = \dot{R}/R$ is the Hubble parameter, H_I^{-1} is the arbitrary time scale characterizing the deflationary period, and $\beta \in [0, 1]$ is a dimensionless parameter of order unity.

Deflationary universe models start their evolution with the largest value of the Hubble parameter, $H = H_I$, for which the phenomenological expression (4) reduces to $\rho_v = \rho_T$ so that we have inflation with no matter and radiation components ($\rho = 0$). At late times ($H << H_I$), we see that $\rho_v \sim \beta\rho_T$, as required by the recent observations. Therefore, if the deflationary process begins at Planck time one has $H_I^{-1} \sim 10^{-43}$s, and since $H_0^{-1} \sim 10^{17}$s it thus follows that $H_0/H_I \sim 10^{-64}$ while the remaining terms are of order unity. Such a conclusion does not change appreciably if the Planck scale is replaced by the grand or even the electroweak unification scales in accordance to the standard model (see paper I). This means that to a high degree of accuracy, the scale H_I is unimportant during the vacuum-dust dominated phase so that for all practical purposes the vacuum energy density can be approximated by $\rho_v = \beta\rho_T$.

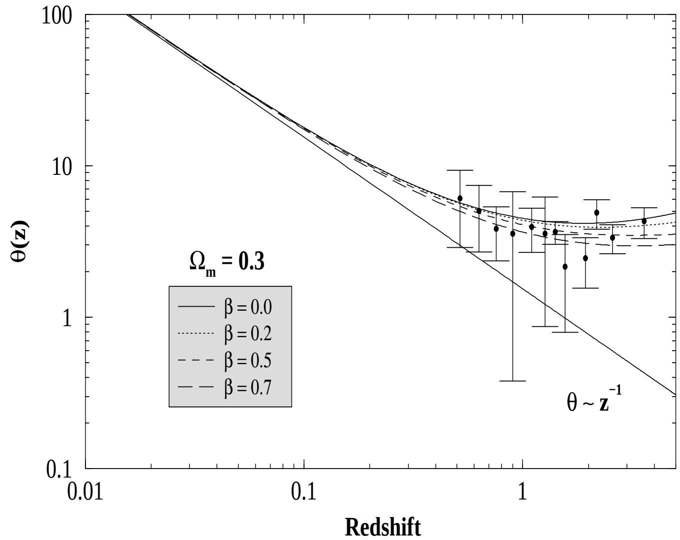

FIG. 1: The angular size - redshift relation for 145 sources binned into 12 bins [25]. The curves correspond to $\Omega_m = 0.3$ and $l = 26.46h^{-1}$ pc.

An interesting feature of decaying $\Lambda(t)$ models is that the temperature dependence on the redshift z of the relic radiation, $T(z)$, can be different from the standard prediction, which is deduced assuming an adiabatic expansion. For deflationary universes, the temperature law at the vacuum/dust phase reads

$$T(z) = T_o(1+z)^{1-\beta}. \quad (5)$$

where T_o is the temperature of the Cosmic Microwave Background (CMB) at $z = 0$. This new relation implies that for a given redshift z, the temperature of the Universe is slightly lower than in the standard photon-conserved scenario. Actually, indirect measurements of $T(z)$ at high redshifts [17, 18, 19] may become one of the most powerful cosmological tests because it may exclude the presence of a cosmological constant or even of any kind of separately conserved quintessence [20].

III. CONSTRAINTS FROM ANGULAR SIZE-REDSHIFT DIAGRAM

The angular size - redshift relation to a light source of proper size l (assumed free of evolutionary effects) can be obtained by integrating the spatial part of Eq. (1) for ξ and ϕ fixed. One finds

$$\theta = \frac{D(1+z)}{\xi(z)} \quad . \quad (6)$$

In the above expression D is the angular-size scale expressed in milliarcseconds (mas)

$$D = 100lh, \quad (7)$$

where l is measured in parsecs (for compact radio-sources). The dimensionless coordinate ξ is given by (see Appendix A of paper I)

$$\xi(z) = \int_{(1+z)^{-1}}^{1} \frac{dx}{x\left[1-(\frac{\Omega_{\rm m}}{1-\beta})+(\frac{\Omega_{\rm m}}{1-\beta})x^{-(1-3\beta)}\right]^{1/2}}. \quad (8)$$

Now, by integrating the previous expression and inserting the result into Eq.(6) one finds

$$\theta = \frac{D\left(\frac{\Omega_{\rm m}}{1-\beta}\right)^{1/2}(1+z)}{\sin\left[\delta sin^{-1}\alpha_1 - \delta sin^{-1}\alpha_2\right]} \quad . \quad (9)$$

where $\delta = \frac{2}{(1-3\beta)}$, $\alpha_1 = (1-\frac{1-\beta}{\Omega_{\rm m}})^{\frac{1}{2}}$, and $\alpha_2 = \alpha_1(1+z)^{-(\frac{1-3\beta}{2})}$.

The above equations show that the predicted value of $\theta(z)$ is completely determined once the values of l, $\Omega_{\rm m}$ and β are given. Two points, however, should be stressed before discussing the resulting diagrams. First of all, the predicted values of $\Omega_{\rm m}$ and β are strongly dependent on the adopted value of l. In the absence of a statistical study describing the intrinsic length distribution of the sources, we consider here the approach recently discussed by Lima and Alcaniz [21]. More specifically, instead of assuming a given value for the mean projected linear size, we work on the interval $l \simeq 20h^{-1} - 30h^{-1}$ pc, i.e., $l \sim O(40)$ pc for $h = 0.65$, or equivalently, $D = 1.4 - 2.0$ mas. In addition, following Kellermann [22], we also assume that compact radio sources are free of the evolutionary and selection effects that have bedevilled attempts to use extended double radio source in this context (see, for example, [23]), as they are deeply embedded in active galactic nuclei, and, therefore, their morphology and kinematics do not depend considerably on the changes of the intergalactic medium. Moreover, these sources have typical ages of some tens of years, i.e., it is reasonable to suppose that a stable population is established, characterized by parameters that do not change with the cosmic epoch [24].

In our analysis we consider the angular size data for milliarcsecond radio-sources recently compiled by Gurvits *et al.* [25]. This data set, originally composed by 330 sources distributed over a wide range of redshifts ($0.011 \leq z \leq 4.72$), was reduced to 145 sources with spectral index $-0.38 \leq \alpha \leq 0.18$ and total luminosity $Lh^2 \geq 10^{26}$ W/Hz in order to minimize any possible dependence of angular size on spectral index and/or linear size on luminosity. This new sample was distributed into 12 bins with 12-13 sources per bin. In order to determine the cosmological parameters $\Omega_{\rm m}$ and β, we use a χ^2 minimization for a range of $\Omega_{\rm m}$ and β spanning the interval [0,1] in steps of 0.02

$$\chi^2(l,\Omega_{\rm m},\beta) = \sum_{i=1}^{12} \frac{\left[\theta(z_i,l,\Omega_{\rm m},\beta) - \theta_{oi}\right]^2}{\sigma_i^2}, \quad (10)$$

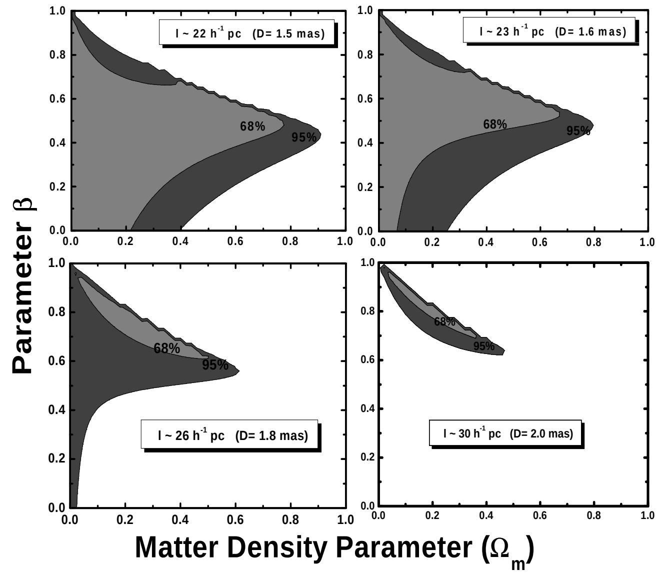

FIG. 2: Confidence regions in the β-$\Omega_{\rm m}$ plane according to the updated sample of angular size data [25]. The solid lines show the 95% and 68% likelihood contours for decaying vacuum models.

D (mas)	lh (pc)	$\Omega_{\rm m}$	β	χ^2
1.5	22.05	0.58	0.58	4.30
1.6	23.53	0.5	0.62	4.32
1.8	26.47	0.34	0.72	4.65
2.0	29.41	0.28	0.76	6.05
Best fit: 1.54	22.64	0.54	0.6	4.28

TABLE I: Angular size constraints on varying Λ cosmologies

where $\theta(z_i,l,\Omega_{\rm m},\beta)$ is given by Eq. (9) and θ_{oi} is the observed values of the angular size with errors σ_i of the ith bin in the sample.

In Fig. 1 we display the binned data of the median angular size plotted against redshift for $\Omega_{\rm m} = 0.3$ and several values of β. The curve for a static Euclidean universe is also shown for comparison ($\theta \sim z^{-1}$). As can be seen from the figure, Euclidean models are strongly deprived by the observational data. In our statistical analysis presented below, it was implicitly assumed that possible selection effects (including the resolution angular limit) are completely under control.

Figure 2 shows contours of constant likelihood (95% and 68%) in the plane $\Omega_{\rm m} - \beta$ for the interval $l \simeq$

$20 - 30h^{-1}$ pc. Note that the allowed range for β is reasonably large unless the characteristic length is also large ($l \sim 30h^{-1}$ pc). For $l \simeq 22h^{-1}$ pc, the best fit occurs for a closed model with $\Omega_{\rm m} = 0.58$ and $\beta = 0.58$. In the subsequent panels of the same figure similar analyzes are displayed for $l \simeq 23.53h^{-1}$ pc (D = 1.6 mas), $l \simeq 26.47h^{-1}$ pc (D = 1.8 mas) and $l \simeq 29.41h^{-1}$ pc (D = 2.0 mas). As physically expected, the limits are now more restrictive for the matter density contribution while larger values of β are allowed. It happens because since $\theta \sim l/\xi$, for the same data (θ_{oi}) and larger l we need larger $\xi(z)$ and, therefore, smaller (larger) values of $\Omega_{\rm m}$ (β). In particular, if we minimize χ^2 for l, $\Omega_{\rm m}$, and β, we obtain $l = 22.64^{-1}$ pc (D = 1.54 mas), $\Omega_{\rm m} = 0.54$, and $\beta = 0.6$ with $\chi^2 = 4.28$ and 9 degrees of freedom. Such values are also in agreement to the ones recently found by Vishwakarma [26]. Indeed, as explained in Paper I, a subclass of the models investigated by him ($\Lambda = n\Omega H^2$) corresponds to the late stages of our deflationary scenario. We also remark that although not discussed here, it is possible to determine exactly the influence of the $\Lambda(t)$-term in the critical redshift z_m at which the angular size takes its minimal value. However, as shown elsewhere [27], the critical redshift cannot discriminate between world models since different scenarios may provide the same z_m values. The main results of this section are summarized in Table 1.

IV. CONSTRAINTS FROM AGES OF GLOBULAR CLUSTERS

Let us now discuss the constraints on deflationary cosmologies related to the expanding age of the Universe. As widely known, a lower bound for this quantity can be estimated in a variety of different ways. For instance, Oswalt *et al.* [28], analyzing the cooling sequence of white dwarf stars found a lower age limit for the galactic disk of 9.5 Gyr. Later on, a value of 15.2 ± 3.7 Gyr was also determined using radioactive dating of thorium and europium abundances in stars [29]. In this connection, the recent age estimate of an extremely metal-poor star in the halo of our Galaxy (based on the detection of the 385.957 nm line of singly ionized ^{238}U) indicated an age of 12.5 ± 3 Gyr [30].

Probably, the most important way of estimating a lower limit to the age of the Universe is dating the oldest stars in GCs. Such estimates have, however, oscillated considerably since the publication of the statistical parallax measures done by Hipparcos. Initially, some studies implied in a lower limit of 9.5 Gyr at 95% confidence level (c.l.) [31], making some authors argue immediately for the end of the age problem [32]. Nevertheless, subsequent studies [33] using new statistical parallax measures and updating some stellar model parameters, found 13.2 Gyr with a lower limit of 11 Gyr at 95% c.l., as a corrected mean value for age estimates of GCs (see also [34]. This implies that the flat cold dark matter (CDM) model is

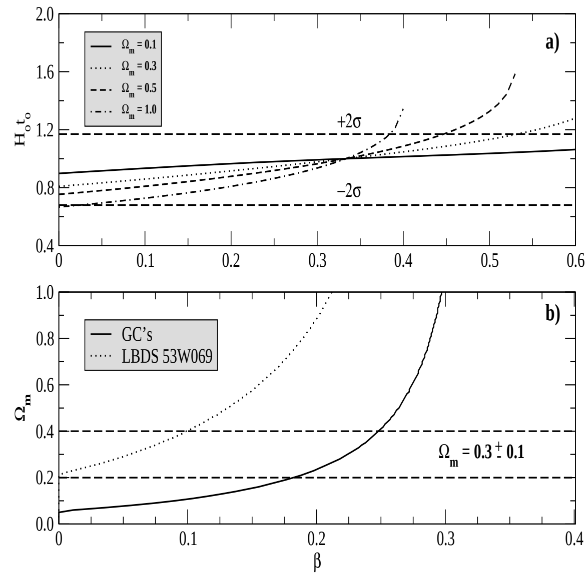

FIG. 3: **a)** $H_o t_o$ as a function of β for some values of the density parameter $\Omega_{\rm m}$. Horizontal lines indicate the $\pm 2\sigma$ limits of the age parameter $H_o t_o = 0.93 \pm 0.12$. **b)** Contours of fixed age parameters. Solid curve corresponds to the median value of the most recent estimates of globular clusters ($t_o = 13 \pm 1$ Gyr) and dashed line to the radio galaxy LBDS 53W069 which has an estimated age of $t_z = 4.0$ Gyr at $z = 1.43$.

ruled out for $h \geq 0.50$, thereby showing that the "age crisis" is not dead at all since the most recent measurements of h point consistently to $h \geq 0.65$ [35, 36, 37]. All these results are also in accordance with recent estimates based on rather different methods for which the ages of the oldest GCs in our Galaxy fall on the interval 13.8 - 16.3 Gyr [38].

The general age-redshift relation for a deflationary universe is (see Paper I)

$$t_o = H_o^{-1} \int_0^1 \frac{dx}{\left[1 - \frac{\Omega_{\rm m}}{(1-\beta)} + \frac{\Omega_{\rm m}}{(1-\beta)} x^{3\beta - 1}\right]^{\frac{1}{2}}}. \qquad (11)$$

As one may conclude, by fixing the $H_o t_o$ from observations, the limits on the β parameter may readily be obtained for some specified values of $\Omega_{\rm m}$. Note also that the age parameter depends only on the product of the two quantities H_o and t_o, which are measured from completely different methods. Here we consider $t_o = 13 \pm 1$ Gyr as a median value for the most recent estimates of globular clusters, and, following Freedman [37], we assume $H_o = 70 \pm 0.7 {\rm Km.s^{-1}.Mpc^{-1}}$. Since these quantities (H_o and t_o) are independent, the two errors may

be added in quadratures providing $H_o t_o = 0.93 \pm 0.12$, a value very close to some determinations based on SN Ia data [2, 39]. From this analysis we see that the Einstein-de Sitter case is off by ~ 2 standard deviations.

In Fig. 3a we show the dimensionless product $H_o t_o$ as a function of β for some values of the density parameter $\Omega_{\rm m}$. Horizontal dashed lines indicate $\pm 2\sigma$ of the age parameter for the values of H_o and t_o considered here. As should be physically expected, the greater the contribution of the vacuum (β) the larger the age predicted by the model. For $\beta = 1/3$, all models predict $H_o t_o = 1$ regardless of the value of $\Omega_{\rm m}$. As explained in Paper I, this critical case corresponds to an expanding universe with constant Hubble flow ($q_o = 0$) also termed "coasting cosmology" [40].

In Fig. 3b we show the plane $\Omega_{\rm m} - \beta$ for the fixed value of the product $H_o t_o = 0.93 \pm 0.12$ (solid curve). Horizontal lines correspond to the observed range $\Omega_{\rm m} = 0.2 - 0.4$ [41], which is used to fix the limits on the β parameter. For this interval we find $\beta \geq 0.18$ and $\beta \geq 0.25$, respectively. For comparison, we also have plotted the curve for an object (LBDS 53W069) with 4.0 Gyr at $z = 1.43$ [42]. The estimated age of this radio galaxy does not provide restrictive limits on the β parameter, thereby showing that deflationary cosmologies are quite efficient to solve the variant of the age crisis at high-z. These results are also in agreement with those found in Ref. [43] in the context of the standard and ΛCDM models.

V. CONCLUSION

We have investigated the observational constraints on deflationary cosmologies provided by the angular size data and age estimates of globular clusters. By considering a sample of 145 milliarcsecond radio-sources recently compiled by Gurvits *et al.* [25] we found that the best fit occurs for a closed model with $\Omega_{\rm m} = 0.54$, $\beta = 0.6$ ($\Omega_{\rm T} = 1.3$), and a characteristic length of $l = 22.64^{-1}$ pc (D = 1.54 mas). As we have seen, angular size measurements from compact radio sources may provide an important test for world models driven by a dark energy component. We stress, however, that constraints from the angular size redshift relation should be taken with some caution because a statistical analysis describing the intrinsic length distribution of the sources is still lacking. This aspect should be specially investigated since the recent development of the observational techniques certainly will provide more accurate data of angular size in the near future.

Concerning the age constraints, by adopting $t_o = 13 \pm 1$ Gyr as a median value for the most recent age estimates of globular clusters and $h = 0.7 \pm 0.07$ we have showed that deflationary models are very efficient to solve the "already" classical age of the Universe problem, as well as its variant related to old galaxies at high redshifts. In particular, regardless of the value of $\Omega_{\rm m}$, for $\beta \geq 0.25$ the class of models studied here provides total ages greater than 14 Gyr.

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